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Abstract—The exponential nature of the rocket equation is well understood in rocket engineering, and empirical data reveals a potentially related exponential relationship between the cost-per-kg of an all-rocket system and a mission's delta-v requirements. The empirical data and assumptions that underpin the empirical relationship are explained. The limitations of the relationship and its utility are discussed.

Keywords—Delta-v, cost of launch

I. INTRODUCTION

Humanity has planned and in many cases launched a wide variety of space missions that have relied entirely on chemical rocket propulsion. Delta-V requirements for these missions range from around 9400 m/s for low-earth orbit to 23,000 m/s for a Mars sample return mission. While plenty of anecdotal evidence suggests that the mission's delta-v requirements significantly affect its overall cost, this paper attempts to quantify the relationship.

By more formally quantifying the cost-versus-delta-v relationship, it becomes easier to answer important policy questions, such as, "How more should I be willing to pay for a 10% improvement on delta-v?" or "Should we add one high delta-v mission to our roadmap or two lower delta-v missions instead?" A more widespread and intuitive understanding of the relationship between delta-v and cost may help people untrained in the science of mission planning, but who are nevertheless involved in shepherding the flow of information, to make better decisions about classifying aspirational plans and capabilities as credible versus misinformation that was manufactured and socialized to generate publicity.

Of course, there are many factors other than delta-v that can affect the cost of a mission, such as whether the mission is crewed or robotic, if the mission involves a small payload or a larger payload, and if the mission is flown frequently or infrequently. Therefore, this study analyzed a wide range of different kinds of missions, and it found that mission cost was strongly correlated with delta-v despite the presence of these other factors.

II. METHODOLOGY

At a high level, the methodology employed here was to find missions where data about the mission's cost and delta-v has been made public by reputable sources. In many cases, the information we sought was available, but not in the exact form that we wanted. For example, in some cases a "C3" value for a mission was provided and we had to use orbital mechanics to convert this information into estimates of the mission's delta-v requirements. In other cases, we found there to be conflicting information on, for example, the actual cost of a launch. The next section, "Empirical Data", has sub-sections that discuss every datapoint that we analyzed and the reasoning behind and decision that we made with respect to that datapoint.

All of the data was captured in a spreadsheet and used to generate an x-y scatterplot with delta-v on the x-axis and cost-per-kg on the y-axis. We used a logarithmic scale on the y-axis and for the x-axis we included in this study two plots - one with a linear scale and the other with a logarithmic scale.

We also fit an exponential trendline to the data and displayed the equation of that trendline on the charts.

Finally, when one or more elements of a mission involved aerobraking, we did a conversion to generate an "equivalent delta-v" value for those elements. See subsection III.B and Appendix A for more information on this conversion.

III. EMPIRICAL DATA

A. Commercial Launches to Low Earth Orbit

If the Earth was an airless world and if the delta-v could be imparted nearly instantaneously, then it would be possible to execute a Hohmann transfer from the surface of the Earth to LEO altitude. In this case, the perigee speed of the Hohmann transfer orbit would be 8000 m/s. An additional 60 m/s of delta-V would be needed to circularize the spacecraft's orbit upon reaching the apogee of the Hohmann transfer orbit. However, if one were to launch east from the Kennedy Space Center, the Earth's rotation would reduce the actual delta-V required by 408 m/s. With these assumptions, the delta-V to LEO can be calculated using orbital mechanics and would work out to be 8000 + 60 - 408 = 7652 m/s. In practice, on a world with an atmosphere, rockets launch vertically to get themselves above the atmosphere before they start accelerating horizontally. It takes them several minutes to accelerate to orbital velocity. Therefore they require extra delta-V to overcome gravity losses and aerodynamic losses. These losses will depend on the rocket, but typically, after gravity and aerodynamic losses are factored in, a rocket needs a delta-V of somewhere between 9000 and 9400 m/s to reach Low Earth Orbit.

The goal here is to determine on average what customers pay to have payloads placed in low earth orbit. SpaceX publishes the price of a Falcon 9 launch on their website and in 2024 the listed price was 69.75 million. The payload to GTO of their highest performing expendable configuration is also published as 8300 kg, but under the listed price there is a note that says, "Up to 5.5 mT TO GTO" which serves to clarify that the price given is not the price for the highest performing expendable configuration[1]. It is most likely that this price is for the lowest-performing configuration – a Return-To-Launch-Site (RTLS) configuration with a short engine bell on the second-stage engine.



Figure 1: Price and payload data for Falcon 9 from SpaceX's website in 2024 showing different GTO values in price and performance sections.

The performance of the RTLS configuration is not published on the website; however, in 2024 SpaceX has posted on Twitter that one of their boosters, B1071, delivered 134 metric tons over 14 missions.



Eight of those missions launched Starlink satellites and landed downrange on a drone ship, and six of the missions returned to the launch site. Two of the missions were ride-share "Transporter" missions. The information in the tweet gives us a rough idea of what the reusable system's average performance is in practice. For this analysis, we shall assume that the Falcon 9 configuration that SpaceX provided a price for, delivers 134,000/14~9500 kgs to low earth orbit. This information allows us to make a cost-per-kg estimate of

$$\frac{69.75M}{9500} = 7342 \text{ USD/kg}$$

It should be noted that there is data that indicates that some customers paid SpaceX more than 69.75M for some of these flights[2], and some of the commercial payloads were sent to higher LEO orbits that require slightly more delta-V to reach. Starlink satellites are generally launched into barely viable[3] orbits that require less delta-v to reach, and they are equipped with Hall-effect thrusters that they use to raise these orbits. This estimate also does not account for insurance, taxes, or any other fees above and beyond the list price.

B. International Space Station (ISS) Resupply

Systems that make round-trips to rotate crews and experiments need to not only accelerate to rendezvous with the station, they also need to *decelerate* to deorbit, reenter, and land safely. The delta-v budget thus increases from 9400 m/s to 17199 m/s (see Table I); however, the aerobraking portion of this budget changes the payload's delta-v using a more efficient technique than chemical rocket propulsion. If we want to properly estimate mission costs, it's not appropriate to include the full aerobraking delta-v in the budget – but it is not appropriate to eliminate it entirely either. Aerobraking systems add mass and the added mass reduces the payload.

TABLE I. DELTA-V FOR ISS RESUPPLY MISSION MANEUVERS

Description of Maneuver	Delta-V (m/s)
Launch to LEO	9400
Raise Orbit Appogee to ISS Altitude	58
Raise Orbit Perigee to ISS Altitide	58
Deorbit Burn	98
Aerobraking ^a	7585
Total	17199

^{a.} Aerobraking is defined here as deceleration where components, such as thermal protection systems, lifting surfaces, or parachutes, interact with the atmosphere to slow the spacecraft.

To achieve this study's goal of determining the relationship between cost and delta-v, we need to delta-v values to be representative of the difficulty of the mission. Giving either zero weight or full weight to aerobraking delta-v's in a mission's delta-v budget would skew the results. To address this issue, we calculated an "equivalent delta-v" for the phases of the mission that involve aerobraking. We estimated the mass of the spacecraft at the beginning of the aerobraking maneuver (m_0), and its mass after subtracting the components needed for aerobraking ($m_{payload}$). This gives us a "payload mass fraction" ($m_{payload}/m_0$). The next step is to imagine that the mass of the non-payload part of m_0 was used to implement a chemical rocket stage that would allow the spacecraft to land instead on an airless moon.

For example, if the payload fraction of a system that takes advantage of aerobraking to land on Mars is equivalent to the payload fraction of a system that uses chemical rockets to land on a small airless moon, then the delta-v value for the small airless moon landing provides us with the "equivalent deltav" value for the Mars landing.

Essentially we are converting a mission made up of a mix of different kinds of delta-v increasing and decreasing maneuvers into a mission where mass is expended at the same rate but where all of the delta-v changing maneuvers are chemical-rocket burns.

The mass of the SpaceX Crew Dragon 2 spacecraft just prior to reentry (m_0) is 21,200 lbs (9616 kg) [4]. We estimated that 15% of the spacecraft's mass is allocated to its Thermal Protection System (TPS)[5] and 10% is allocated to its parachute system. Subtracting the mass of those systems leaves us with 7212 kg - an amount considerably larger than the spacecraft's rated downmass of 2507 kg[6]. So we still need to account for 7212-2507=4705 kg of mass. Some of this will be structure, the reaction control systems, the Draco engines used for in-space maneuvers and launch abort, and propellant. Other than some of the structure, these components are not needed to keep the crew alive. Let's assume that 2000 kg of the remaining 4705 kg is needed for communications systems, crew couches, environmental control systems and the structure of the pressurized compartment. This gives us a value for m_{pavload} of 4507 kg. We can then use the following formula (see Appendix A for its derivation) to estimate the equivalent delta-v associated with the payload ratio.

$$\Delta V = V_e \ln \left(\frac{m_0}{m_{payload}} \div \left(\frac{m_0}{m_{payload}} k_1 + 1 \right) \right)$$
$$\Delta V = 3270 \ln \left(\frac{9616}{4507} \div \left(\frac{9616}{4507} 0.05 + 1 \right) \right) = 2147 \ m/s$$

The delta-V budget with the equivalent aerobraking delta-v value is shown in Table II.

TABLE II. EQUIVALENT DELTA-V FOR ISS RESUPPLY

Description of Maneuver	Equivalent Delta-V (m/s)		
Launch to LEO	9400		
Raise Orbit Appogee to ISS Altitude	58		
Raise Orbit Perigee to ISS Altitide	58		
Deorbit Burn	98		
Aerobraking (using Equivalent Delta-V)	2147		
Total	11761		

The cost-per-kg for ISS resupply was covered in an earlier article[7] which discussed four reputable sources of information. These sources were in rough agreement with one another and placed the cost-per-kg for ISS resupply at around 80,000 USD/kg.

C. One-way Trip to Low Lunar Orbit

The Delta-V required to place a spacecraft in low lunar orbit was estimated, by tabulating values on a delta-v map, to be 13340 m/s. This destination does not involve any aerobraking maneuvers.

Astrobotic, a company with expertise in lunar landers and rovers, published a Payload User Guide that states on page 5, "Companies, governments, universities, non-profits, and individuals can send payloads to the Moon at \$300K, \$1.2M, or \$4.5M per kilogram of payload delivered to lunar orbit, to the lunar surface, or on a rover, respectively."[8] From this statement, we can obtain the cost-per-kg to be 300,000 USD.

D. One-way Trip to the Surface of the Moon

The Delta-V required to land a spacecraft on the moon was estimated, by tabulating values on a delta-v map, to be 15060 m/s. This destination does not involve any aerobraking maneuvers.

From Astrobotic's statement (see previous section) we can obtain the cost-per-kg to be 1,200,000 USD.

E. Europa Clipper with Falcon Heavy

NASA's Europa Clipper mission to Jupiter launched on an expendable version of the Falcon Heavy on October 14th, 2024. According to Jon Edwards, VP of Falcon Launch Vehicles at SpaceX, the launch set a "new Falcon speed record: 12680 m/s (earth-centered inertial)"[9] This value agrees well with the C₃ value of 41.71 km²/s² that was called for in the 2024 Mars-Earth Gravity Assist (MEGA) mission plan[10]. The square root of the C3 value is the excess velocity which can be converted to a perigee value with the formula

$$v_p = \sqrt{\frac{2g_E}{R_E + h} + \frac{g_E}{|a|}}$$

Where

 v_n is the perigee velocity

 g_E is the Earth's gravitational parameter

 R_{E} is the Earth's radius

'h' is the altitude of the hyperbolic trajectory's perigee

'a' is the hyperbolic trajectory's negative semi-major axis

This can be calculated with the formula

$$a=-g_E C_3$$

If we plug in a value of 200 km for 'h', these above formulas calculate a perigee velocity of 12766 m/s.

A query using NASA's Launch Vehicle Performance Website[11] shows that the expendable version of Falcon Heavy is, at least in theory, capable of launching a 6400 kg payload to a C_3 of 41.71 km²/s².

The perigee velocity of the launch trajectory is roughly 4880 m/s greater than the LEO orbital velocity of 7800 m/s. To more accurately estimate the total delta-v, we will substitute the 7800 m/s value with the delta-v-to-LEO value of 9400 m/s that we used earlier. This gives us an estimated overall delta-v for the launch of 9400 + 4880 = 14280 m/s.

Incidentally, Falcon Heavy's second stage did a 3-minute and 21-second burn while in its parking orbit to accelerate by 4880 m/s. The burn placed the spacecraft on an Interplanetary Transfer orbit.



Figure 3: Falcon Heavy Performance from NASA Website

To achieve this feat, SpaceX expended its two side-boosters in addition to the core and second stages, which are normally expended on Falcon Heavy flights. The side boosters are normally reused, but this was the pair's sixth and final mission.

Europa Clipper's dry mass is 3241 kg and its propellant mass is 2750 kg for a total of 5991 kg. On Jul 23, 2021, NASA announced that they had awarded the contract for launch services to SpaceX. According to usaspending.gov, the Current Award Amount is 179.4 million USD[12]. Thus, the cost per kg for this mission was 179.4e6/5991=29945 USD/kg. It should be noted, however, that because the side boosters had been reused five times before, NASA's negotiators may have married the expendable configuration's performance with the reusable configuration's cost. In a 2019 report from the Office of the Inspector General, the expendable Falcon Heavy configuration (presumably new) was estimated to cost up to 450 million[13]. Some of the earlier and partially reusable Falcon Heavy missions were more expensive, but these missions also included the cost of one-time upgrades to the Falcon Heavy system.

F. Europa Clipper with SLS

SLS would have offered a faster ride for Europa Clipper. A SpaceNews article stated, "What is not in doubt, though, an SLS launch would have allowed the spacecraft to fly directly to Jupiter, arriving less than three years after launch. With Falcon Heavy, Europa Clipper will make gravity-assist flybys of Mars and Earth, arriving at Jupiter five and a half years after launch."[14] The delta-V required for a Jupiter transfer orbit is 15300 m/s. The marginal cost of an SLS launch was estimated to be 876 million USD, although the marginal cost is not the same as the actual contract cost. Some sources place the actual cost of an SLS launch as high as 4 billion dollars. We included this datapoint at the rate that NASA's Office of the Inspector General published because there was a some competition between SpaceX and SLS to launch the Europa Clipper mission, and this price is representative of what SLS's "bid" was.

G. SLS Perfomance Curve

Payload versus C3 data for SLS can be found in the SLS Mission Planner's Guide[15]. The C3 values were converted to delta-v values using the technique explained in Section E.

To represent the cost as a function of delta-v we assumed a price-per-launch of 2.5 Billion.

H. Falcon Heavy Expendable Performance Curve

Payload versus C3 data was obtained from the green curve in Figure 3 and converted to delta-v values using the technique explained in Section E. The cost of an expendable launch is equivalent to what NASA paid SpaceX to launch the Europa Clipper mission.

I. One-way trips to the surface of Mars

The Delta-V required to land a spacecraft on Mars was estimated, by tabulating values on a delta-v map, to be 18510 m/s; however, 5910 m/s of that is deceleration that can be achieved through aerobraking during Entry, Descent, and Landing (EDL). A variety of different EDL systems have been proposed (and some implemented) for missions to Mars.

The Mars 2020 spacecraft, which carried Perseverance and Ingenuity, had a mass of 3649 kg when it departed from earth at a speed of 11028 m/s after being launched by an Atlas V 541, but its rover and helicopter together mass just 1025 kg. This gives us a rough estimate of the initial mass (m_0) and the payload mass (mpayload). With this information we can calculate an equivalent delta-v as we did earlier in the ISS resupply section.

$$\Delta V = 3270 \ln \left(\frac{3649}{1025} \div \left(\frac{3649}{1025} 0.05 + 1 \right) \right) = 3616 \ m/s$$

According to delta-v maps the delta-V between the surface of Mars and MTO is 5910 m/s so the Mars 2020 EDL system generated about 1.8 times more delta-v than a chemical rocket would with the same payload fraction would have.

Component	Mass (kg)	Mass Percentage
Cruise Stage	539	14.77%
Aeroshell	575	15.76%
Descent Vehicle Dry Mass	589	16.14%
Heat Shield	440	12.06%
Parachute	81	2.22%
Landing Propellant	400	10.96%
Rover (Perseverance)	1023.2	28.04%
Helicopter (Ingenuity)	1.8	0.05%
Total	3649	

TABLE III. MARS 2020 SPACECRAFT MASS BREAKDOWN

The equivalent delta-v for this mission is 18910 - 5910 +3616 = 16616 m/s.

Total

The launch costs for the mission were approximately 243 million, according to a NASA press release[16], "which includes: the launch service; spacecraft and spacecraft power source processing; planetary protection processing; launch vehicle integration; and tracking, data and telemetry support." They do not cover the cost of the spacecraft or the EDL system. According to a NASA press kit, "NASA has invested approximately \$2.444 billion to build and launch the Mars 2020 Perseverance mission."[17][18] The total cost of the mission needs to be divided up between the cost of the payload and the cost of everything that was used to deliver the payload to the destination. Furthermore, the argument could be made that even some of the payload's costs, such as costs associated with making the payload sufficiently lightweight and robust, are, indirectly, delivery costs. For example, an Office of the Inspector General's report on NASA'S Mars 2020 Project stated that "Mars 2020 managers have identified rover mass growth as one of the Project's open risks." And "if necessary, they could take additional steps such as removing a proposed helicopter technology demonstration from the mission to keep the mass below 1,050 kg."

Because we know the mass of the rover and helicopter make up 30% of the dry mass of the entire Mars2020 spacecraft, we start with a baseline assumption that the costs are portioned in a similar manner and adjust from there. Nonpayload parts of the spacecraft drew heavily from heritage technologies used on earlier missions, so this would cause us to adjust the 30% value upwards. Accounting for funds spent to reduce the mass of the rover would lead us to adjust the 30% value downward. For this study, we will assume that these two effects roughly cancel out; therefore 30% of the cost of the spacecraft was related to engineering the payloads to explore the surface of Mars and 70% of the cost was for work that made it possible to deliver the payloads to the surface of Mars.

The total cost of delivery is thus the launch costs (\$286 million in 2024 USD) plus 70% of the formulation and implementation costs (\$3232 million in 2024 USD) for a total of \$2.62 billion in 2024 USD.

J. Apollo

Apollo was both a flags-and-footprints mission as well as a sample-return and the mission design was, therefore, very focused on the "round-trip" aspect of the mission.

The delta-v required for the outward bound trip to the surface of the moon is 14660 m/s. The return trip requires 2540 m/s of acceleration delta-v and as additional 12120 m/s of [19]deceleration delta-v that can take advantage of aerobraking. To convert the aerobraking part of the mission into a payload-ratio equivalent chemical rocket delta-v, we need to first determine m_0 and m_p . The command module's mass when it separated from the service varied slightly from mission to mission, but was on average, 5672 kg (page 307 of [20]). To obtain m_p we need to divide up this mass into components and systems that were used for entry, descent, and landing, and parts to classify as mission payloads.

Let's assume the lunar samples, astronauts, and parts of spacecraft that they depend on for life support are considered payload. The heat shield, parachutes, propellant, and reaction control thrusters will be treated as systems that are either consumed or discarded during or after reentry. Even if some of these parts did in fact end up in a museum, we will still treat them as mass that would not exist if the mission were instead designed to end with a landing on an airless moon.

TABLE IV. APPOLO COMMAND MODULE MASS BREAKDOWN*

Component	Mass (kg)	EDL Fraction	Payload Fraction
Structure	1560	0.5	0.5
Heat Shield	848	1.0	0
RCS Engine	33	1.0	0
RCS Propellant	120	1.0	0
Recovery Equipment	240	1.0	0

Component	Mass (kg)	EDL Fraction	Payload Fraction
Main and Drogue Parachutes	334	1.0	0
Navigation equipment	505	1.0	0
Telemetry equipment	200	1.0	0
Electrical equipment	700	0.5	0.5
Communications systems	100	0	1.0
Crew couches and provisions	550	0	1.0
Environmental Control System	200	0	1.0
Crew and Collected Samples	282	0	1.0
Total	5672 kg	3410 kg	2262 kg

* The values in this table should be treated as best-effort estimates resulting from internet sleuthing – they are not official numbers from a highly reputable source.

From Table IV, we arrive at an estimate for $m_{payload}$ of 2262 kg. To determine the equivalent delta-v for a rocket stage with the same payload mass ratio, we use the formula derived in Appendix A.

$$\Delta V = 3270 \ln\left(\frac{5672}{2262} \div \left(\frac{5672}{2262} 0.05 + 1\right)\right) = 2620 \ m/s$$

Thus the total equivalent delta-v for the mission is 14660 + 2540 + 2620 = 19820 m/s.

The payloads for the mission is fundamentally astronauts that walked on the moon and samples from the moon. As President Kennedy put it, "I believe that this nation should commit itself to achieving the goal, before this decade is out, of landing a man on the moon and returning him safely to the earth." The lunar samples that were collected were also of immense scientific value - they confirmed that the Moon's origin as a result of a giant impact with early Earth. Ground personnel, backup crews, and astronauts that did not reach the surface of the moon were all critical to the success of the mission, but did not add to the total of number of people who walked on the moon. The total mass of the 12 astronauts who walked on the moon was 907 kg at the start of the mission. The total mass of all of the lunar samples that they brought back was 382 kg. Therefore the total payload mass for the Apollo space program was 1289 kg. A published analysis placed the cost at 257 billion in inflation adjusted to 2020 USD[21]. We adjusted this value to 2024 USD to arrive at an updated value of 307 billion.

K. Mars Sample Return

The Delta-V requirements for a basic sample return mission are estimated to be 20661 m/s. To arrive at this value

the following delta-v values from a delta-v map were used: 9400, 3210, 390, 2110, and 3800 m/s. On the outbound journey the 2110 m/s and 3800 m/s values were replaced with the equivalent delta-v value of 3318 m/s calculated in the Mars2020 section. On the return journey the 9400 m/s and 3210 m/s values were replaced with the equivalent delta-v value of 1969 m/s calculated in the Mars2020 section.

An Internal Review Board report on the Mars Sample Return program stated "Alternate architectures should be examined under clear guidelines provided by NASA HQ for yearly budget constraints, while acknowledging that the lifecycle cost will likely be in the \$8 to \$11B range regardless of architectural choices."[22] At the 2024 Mars Society Conference, a research familiar with the program said that the proposed architecture involved returning 30 sample tubes, weighing 80 grams each. Therefore we shall assume that the payload for this mission will be 2.4 kg.

The estimates for this datapoint are forward looking and thus subject to more uncertainty than an estimate for a past program with lots of publicly available data such as the Apollo program. Expensive space programs have a history of costing more than initially estimated. NASA invited the commercial sector to propose more cost-effective mission architectures and this could lead to lower costs; however, if those architectures propose to bring back less mass, then the costper kg could remain high.

IV. RESULTS

The results from the previous sections are shown in Figure 4 and Figure 5. The blue line is an exponential trendline which has been fit to the data. The equation of this trendline is

$$CostPerKa = 0.30481 e^{9.99468 \times 10^{-4} \Delta V}$$

...where ΔV is measured in m/s. However, a potentially more useful "rule of thumb" would be

$$CostPerKg \cong 0.3 e^{\Delta V}$$

...where ΔV is the "equivalent delta-v" measured in km/s.

A slightly less accurate but more timeless version of the above rule-of-thumb is

$$CostPerKg \cong \frac{CPI}{1000}e^{\Delta V}$$

...where CPI is the Consumer Price Index.



Figure 4: Launch Costs Versus Mission Equivalent Delta-V



Figure 5: Launch Cost Versus Mission Equivalent Delta-V on a log-log chart

V. DISCUSSION

Figure's 4 and 5 show that the does appear to be an exponential relationship between Equivalent Delta-V and cost-per-kg. However, this an empirically observed relationship. A more sophisticated analysis would show cost-per-kg as a function of more parameters, such as mission payload and year of mission. It seems logical that a mission with a larger payload would be better able to take advantage of economies of scale, and that a mission with a more recent launch date would be better able to take advantage of technological advantages.

We do observe, for example, that the SLS and Falcon Heavy performance curves are slightly below and to the right of the curve of best fit. This implies that newer technologies perform significantly better than older technologies, such as Apollo's Saturn V rocket. However, The Apollo data point factored in the cost of test missions and one failed mission. Apollo was also a human-rated mission which means that the engineers would have used the system more conservatively – to leave some margin between the delta-v that it could have achieved in theory and the delta-v that is achieved in practice. The data sources used for SLS and Falcon Heavy report theoretical best case performance numbers without including extra margin that most missions plans would prefer to factor in.

Another important consideration is that the payload numbers for SLS and Falcon Heavy do not include the need for spacecraft mass that guides the payload towards its intended target. Almost every mission (excepting perhaps the launch of the Tesla Roadster) requires a propulsive stage that keeps the probe or lander on course for the planet or asteroid that it's going to. The mass of this propulsive stage really should be subtracted from the payload. This would in effect raise the cost-per-kg of the non-propulsive part of the payload and move these curves closer to the fitted cost curve.

The performance curve for the fully-expended¹ Falcon Heavy shows that it performs better on cost-per-kg than the partially-reusable Falcon Heavy (see Figure 6).



Figure 6: Cost curves for SLS, expendable Falcon Heavy, and partially reusable Falcon Heavy.

While this may be true, it is hard to reconcile that conclusion with the fact that most Falcon Heavy missions are of the partially-reusable variety. It is more likely that SpaceX made an aggressive bid to win the expendable Falcon Heavy contract – possibly because there was some strategic value to out-competing SLS for this mission. SLS had more political support, and could have placed Europa Clipper on a trajectory that would have it arrive at Jupiter much sooner by avoiding the need for multiple inner-planet flybys to pick up extra delta-v.

If we increase the price of the expendable Falcon Heavy from the Europa Clipper cost of 179.4 million to 750 million its performance curve lines up better with the SLS and partially reusable Falcon Heavy curves (see Figure 7). What this means is if we didn't know the price of the expendable version of Falcon Heavy, and we used the cost curve to estimate what its price should be, our estimate would have been higher than the actual cost by a factor of 4.



Figure 7: Cost Curves if the price of the expendable version of Falcon Heavy is assumed to be 750 million instead of 179.4 million.

Nevertheless, it remains difficult to explain how the partially-reusable version of Falcon Heavy is a better performing technology on a cost-per-kg basis than the expendable version of Falcon Heavy. Reusability may not be a particularly cost-effective technology for the missions in

¹ Almost fully-expended – the fairings were recovered.

the market that Falcon Heavy normally serves. This hypothesis is supported by the fact that SpaceX has, so far, apparently not prioritized the operational status of an even more reusable Falcon Heavy configuration where the core stage (in addition to the side boosters) is reused.

APPENDIX A – THE EQUIVALENT DELTA-V FORMULA

The rocket equation is very useful because it is simple, intuitive, and it communicates that there's an exponential effect in play, but it does not provide an easy way to convert a payload ratio into a delta-v value. Therefore, we need to derive an equation that will allow us to do this. Considerer that the final mass, m_f , in the rocket equation includes, for example, the mass of:

- An engine or engines capable of lifting the initial mass, m₀, off the launch pad,
- Tanks for holding the propellants,
- Avionics hardware,
- Any systems and propellant relevant to recovering and landing a spent rocket stage,
- Various structural and plumbing elements, and
- The payload.

The mass of many of these components scales linearly with the initial mass, m_0 , so we can further define m_f as

$$m_{f} = m_{variable} + m_{fixed} + m_{payload}$$
$$m_{f} = k_{1}m_{0} + m_{fixed} + m_{payload}$$

Where:

 k_1 is the ratio of the mass of the variable parts of the rocket (those that scale linearly with the rocket's initial mass) to the initial mass of the rocket.

' m_{fixed} ' is the mass of the components that do not scale with the size of the rocket, such as, for example, an avionics component like a flight computer.

Now we can substitute our expression for m_f into the rocket equation...

$$\mathbf{m}_0 = (\mathbf{k}_1 \mathbf{m}_0 +) \mathbf{e}^{\frac{\Delta V}{V_e}}$$

Let's substitute

$$m_{f\&p} = m_{fixed} + m_{payload}$$

And rearrange...

$$\begin{split} m_0 &= k_1 m_0 e^{\frac{\Delta V}{V_e}} + m_{f\&p} e^{\frac{\Delta V}{V_e}} \\ m_0 \left(1 - k_1 e^{\frac{\Delta V}{V_e}}\right) &= m_{f\&p} e^{\frac{\Delta V}{V_e}} \\ \frac{m_0}{m_{f\&p}} &= \frac{e^{\frac{\Delta V}{V_e}}}{1 - k_1 e^{\frac{\Delta V}{V_e}}} \end{split}$$

This gives us an equation that relates the initial mass of the rocket to just the mass of the payload and the fixed mass components, such as some avionics components.

We can rearrange the equation as follows if we want to calculate delta-v from the payload ratio.

$$\begin{split} \frac{m_0}{m_{f\&p}} &= \frac{m_0}{m_{f\&p}} k_1 e^{\frac{\Delta V}{V_e}} + e^{\frac{\Delta V}{V_e}} \\ \frac{m_0}{m_{f\&p}} &= \left(\frac{m_0}{m_{f\&p}} k_1 + 1\right) e^{\frac{\Delta V}{V_e}} \\ \frac{\Delta V}{V_e} &= \ln\left(\frac{m_0}{m_{f\&p}} \div \left(\frac{m_0}{m_{f\&p}} k_1 + 1\right)\right) \\ \Delta V &= V_e \ln\left(\frac{m_0}{m_{f\&p}} \div \left(\frac{m_0}{m_{f\&p}} k_1 + 1\right)\right) \end{split}$$

Where

'Ve' is the exhaust velocity of the rocket

'm₀' is the initial mass and includes everything

' $m_{f\&p}$ ' is the mass of the payload plus the mass of non-payload components that do not scale with the initial mass of the rocket.

'k1' is a type of "structural mass" coefficient.

The m_{fixed} part of $m_{f\&p}$ can be assumed to be negligibly small for larger rockets and for modern rockets that benefit from modern microelectronics. In this paper we will assume that $m_{f\&p} = m_{payload}$

Because cryogenic propellants tend to boil off, a common choice for propellants when the stage needs to be used days or weeks after the start of the mission is the combination of hydrazine (N₂H₄) plus nitrogen tetroxide (N₂O₄). These are hypergolic propellants which achieve an ISP of ~334 s [23], which equates to an exhaust velocity (V_e) of roughly 3270 m/s. For comparison, cryogenic propellants have theoretical maximum exhaust velocities in a vacuum of 3615 m/s for Methalox and 4462 m/s for Hydrolox.

The structural mass coefficient (k_1) for a very-wellengineered rocket stage is around 0.05. If you imaging trying to build a racecar capable of accelerating up a steep slope at 1.5 g's while carrying 19 times its own weight in fuel, oxidizer, and payload, you will gain some appreciation for how impressive rocket engineering is.

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